Q1)

Given that

a) The tangent plane at the point is

b) Since , therefore is unit vector. Then, the directional derivative is given by

Q2

By Lagrange multipliers for the constrains , we have

Substitute and into , we get

With , it holds that and

With , it holds that and

Thus, the maximum value with the given constrain is happens at and the minimum value is happens at

Q3

a)

The volume of the given solid is given by

With the Riemann sum:

Then the Riemann sum approximation for is

b)

Let , we obtain , therefore, the region is

Since change the sign when is passing 0. Therefore, The volume of the given solid is given by

4)

a) We have

thus, , which means that is conservative field.

b) Since, we have

Therefore

From

Comparing that result with gives us

therefore, . Thus, chose , we obtain a particular potential field

c)

From , Let , for , Then

Then

Q5)

a)

b)

We have

For upward orientation, we have